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SIXTH SEMIANNUAL STATUS REPORT

OF

NASA RESEARCH GRANT NO. NsG-568

Title of Project: Stochastic Models for Multi-valued, Multi-dimensional Relations

Principal Investigator: Triloki N. Bhargava
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Kent State University, Kent, Ohio

Period Covered: June 16, 1966 to January 15, 1967.

Date: March 11, 1967

Signature of Principal Investigator T. N. Bhargava

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PRELIMINARIES

This is the sixth semiannual status report of the NASA Research Grant Number NsG-568, covering the period from June 16, 1966, to January 15, 1967, of the research project entitled "Stochastic Models for Multi-valued, Multi-dimensional Relations". Semiannual status reports covering the period up to June 16, 1966, have been submitted earlier.

It is hoped that the project work will be concluded by the end of September, 1967, when a detailed report will follow including a research monograph by the principal investigator on stochastic models.

For the sake of convenience, clarity and continuation, this report is divided into three parts.

I. Product

1. Research Papers
2. Theses
3. Professional Meetings
4. Visiting Mathematicians
5. Graduate Students

II. Chronological Summary

III. Current Research Activities

1. Covered Spaces
2. Extension of Riemmanian Geometry
3. Maximal Functions over Partitions

Resumés

I. PRODUCT

The principal accomplishments made under NASA Research Grant Number NsG-568 over the total duration of the grant to date are summarized in the five categories below, each of which has been organized chronologically when feasible. Starred (*) items have not been presented in previous semiannual status reports. SASR is used as an abbreviation for Semiannual Status Report.

1. Research Papers:

(i) Bhargava, T. N., "Time Changes in a Digraph"; Journal of Applied Probability, June, 1965, (Technical Report No. 3, SASR No. 2).

(ii) Chatterji, S. D., "Measures Induced by Continued Fractions"; Mathematische Annalen (in German), 1966, (Technical Report No. 7, SASR No. 3).

(iii) Chatterji, S. D., "An Inequality for Poisson Distributions"; Submitted to Annals of Mathematical Statistics, (Technical Report No. 11, SASR No. 3).

(iv) Bhargava, T. N. and Ahlborn, T. J., "On Topological Spaces Associated with Digraphs"; Submitted to Acta Mathematica, also NASA CR-56157, (Technical Report No. 1, SASR No. 2).

(v) Chatterji, S. D., "On Counting Topologies"; Submitted to American Mathematical Monthly, (Technical Report No. 4, SASR No. 2).

(vi) Bhargava, T. N. and Uppuluri, V. R. Rao, "On Certain Biases in Voting Problems"; To be submitted to American Statistician, (Technical Report No. 6, SASR No. 3).

(vii) Bhargava, T. N. and Ohm, S. E., "An Ideal Topology for General Binary Systems"; Submitted to Czechoslovakia Journal of Mathematics, (Technical Report No. 10, SASR No. 3).

(viii) Fisk, D. L., "Sample Quadratic Variation of Sample Continuous, Second Order Martingales"; To be published in Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gabiete, (Technical Report No. 12, SASR No. 5).

(ix) *Bhargava, T. N. and O'Korn, L. J., "On Graphs with Minimum Elongation Diameter"; Submitted to Israel Journal of Mathematics, (Technical Report No. 15).

(x) Bhargava, T. N. and Wilson, M. C., "Entropy and Random Graphs"; To be submitted to Information and Control, (Technical Report No. 16, SASR No. 5).

(xi) *Bhargava, T. N. and O'Korn, L. J., "On Bratton's Conjecture and a Theorem of Luce"; Submitted to Portugaliae Mathematica, (Technical Report No. 17).

(xii) *Doyle, P. H., "On Finite T_0-Spaces"; To be published in Proceedings of the Prague Second Symposium on Topology, (Technical Report No. 18).

(xiii) *Doyle, P. H., "On Shrinking Arcs in Metric Spaces"; (Previously titled "Counterimages of Arcs"), To be published in Israel Journal of Mathematics, (Technical Report No. 19).

(xiv) *Bhargava, T. N., "On Number of Subdigraphs of Order 2 and 3 with Connectedness Properties"; To be submitted to Canadian Mathematics Journal, (Technical Report No. 20).

(xv) *Chatterji, S. D., "Comments on the Martingale Convergence Theorem"; Submitted to Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gabiete, (Technical Report No. 21).

2. <u>Theses</u>

(i) Ahlborn, T. J., "On Directed Graphs and Related Topological Spaces", August, 1964, (Technical Report No. 2, SASR No. 2).

(ii) Ohm, S. E., "Some Topological Properties of Halfgroupoids", August, 1965, (Technical Report No. 8, SASR No. 3).

(iii) Wilson, M. C., "On Certain Information-Theoretic Concepts in the Theory of Graphs", August, 1965, (Technical Report No. 9, SASR No. 3, NASA CR-478).

(iv) O'Korn, L. J., "Diameters of Graphs and Digraphs", August, 1966, (Technical Report No. 13, SASR No. 5).

(v) Piotrowski, R. W., "On Application of Hasse Diagrams for Counting Topologies on Finite Spaces", August, 1966, (Technical Report No. 14, SASR No. 5).

(vi) *Edelman, J. E., "On Covered Spaces", in preparation, (see III.1).

(vii) *Smucker, R. A., "On Maximizing a Function over Partitions", in preparation, (see III.3).

3. <u>Papers Presented at Professional Meetings</u>:

(i) Bhargava, T. N., "Digraph Topology", <u>American Mathematical Society meetings</u> in Miami, <u>Florida</u>, January, 1964, (SASR No. 1).

(ii) Bhargava, T. N., and Ahlborn, T. J., "Directed Graphs and Point-Set Topology", <u>American Mathematical Society meetings</u> in New York, April, 1964, (SASR No. 2).

(iii) Bhargava, T. N. and Chatterji, S. D., "Random Graphs and Digraphs", at <u>Joint European Conference of the Institute of Mathematical Statistics</u> at Berne, Switzerland, September, 1964, (<u>Invited</u> paper), (SASR No. 2).

(iv) Bhargava, T. N. and Ohm, S. E., "Some Results on Algebraic Structures and the Digraph Topology", (By title), American Mathematical Society meetings in New York, April, 1965, (SASR No. 5).

(v)*Chatterji, S. D., "Comments on the Martingale Convergence Theorem", NATO Conference on Probability and Statistics, Loutraki, Greece, May-June, 1966.

(vi)*Doyle, P. H., "On Finite T_0-Spaces" (Presented by Bhargava) Second Symposium on Topology and its Relations to Modern Analysis and Algebra, Prague, Czechoslovakia, September, 1966.

4. Visiting Mathematicians Supported by NASA:

(i) Doyle, P. H., Professor of Mathematics, Michigan State University. Visited July, 1966; August 1-6, 1966.

(ii) Fisk, D. L., Assistant Professor of Mathematics, Northwestern University. Visited October 21-23, 1966; December 19-21, 1966.

(iii) Gani, J., Chairman, Department of Statistics, University of Sheffield, England. Visited July 28-30, 1966.

5. Graduate Students Supported by NASA:

(i) Ahlborn, T. J., M.A., Kent State University, (August, 1964); Working on Ph.D. as a graduate assistant in Department of Mathematics, University of Rochester.

(ii) Curtis, C. C., B.S.Ed., Kent State University; Teaching in the public school system, Kent, Ohio.

(iii) Edelman, J. E., B.A., Kent State University; M.A. in progress at Kent State University.

(iv) Flagg, B. O., B.S., LeMoyne College; Awarded NDEA fellowship for pre-doctoral study in Department of Statistics, University of Minnesota.

(v) Ohm, S. E., B.S.Ed., Kent State University; M.S., Kent State University, (August, 1965); Awarded NASA traineeship for pre-doctoral study in mathematics, Purdue University.

(vi) O'Korn, L. J., B.S.Ed., Clarion State College; M.A., Kent State University, (August, 1966); Awarded a fellowship from the Department of Mathematics, Ohio State University for pre-doctoral study.

(vii) Piotrowski, R. W., B.S., Alliance College; M.A., Kent State University, (August, 1966); Awarded assistantship for pre-doctoral study in mathematics, Virginia Polytechnic Institute.

(viii) Rohatgi, V. K., B.Sc. and M.A., Delhi, India; M.Sc., University of Alberta; Presently enrolled jointly at Michigan State University and the University of Sheffield as a doctoral student and should receive Ph.D. in 1967.

(ix) Smucker, R. A., B.S., Goshen College; M.A. in progress at Kent State University.

(x) Watters, H. H., B.S., Morehouse College; Course work for M.A. degree completed at Kent State University; Presently working for Eastman Kodak, Rochester, N. Y. and working on thesis requirement for M.A.

(xi) Wilson, M. C., B.S., Presbyterian College; M.A., Kent State University, (August, 1965); Instructor at Kent State University and working parttime for doctorate at Western Reserve University.

II. CHRONOLOGICAL SUMMARY

The following represents a summary of work done during this report period, divided on the basis of academic quarters.

June 16 to September 15, 1966: Research assistants O'Korn and Piotrowski were fully supported by NASA during this period and both completed their thesis requirement for a masters degree. The principal investigator, jointly with O'Korn, completed two research papers as of their work in graph theory (Technical Reports Nos. 15 and 17). Doctoral student Rohatgi (of Michigan State University and the University of Sheffield) joined the project for July and most of August under partial support of NASA. His work with the principal investigator in the field of combinatorial mathematics has been most beneficial to the project. Rohatgi is expected to complete his requirements for a doctorate degree in June, 1967. Professor P. H. Doyle of Michigan State University, the well-known topologist and a research consultant on the project, visited Kent for further research work and consultation during July and again for a week in August. Doyle was quite interested in the work of graduate students on the project and consulted with them individually during his visits. Professor Doyle wrote two papers (Technical Reports

Nos. 18 and 19), one for the Prague Symposium and the other for publication in the Israel Journal of Mathematics.

Professor J. Gani, Chairman of the Department of Statistics, the University of Sheffield, visited Kent during July for research consultation with the principal investigator. Research associate Wilson continued on the project throughout the summer working in the field of information theory in relation to graph theory. Graduate students Watters and Edelman were partially supported by the project during this period. Watters worked on his thesis requirement for a masters degree, and Edelman, a second year graduate student in the department, began research for his thesis in the area of graph theory and topology.

During the period from August 15 to September 12, the principal investigator attended professional meetings of the International Mathematical Congress at Moscow University and the "Second Symposium on General Topology" held at Matematický ústav CSA in Prague, Czechoslovakia. At the Prague meetings Bhargava presented a paper by P. H. Doyle entitled "On Finite T_0-Spaces". The principal investigator also visited the Institute of Mathematical Statistics in Copenhagen, Denmark, where research associate Chatterji is a visiting professor.

September 16, 1966 to January 15, 1967: At the beginning of the fall quarter at the university, graduate students Edelman and Smucker became fulltime research assistants on the project. Both are second year graduate students in the department and plan to complete their course and thesis requirements for graduation with a masters degree. Several other graduate students have expressed interest in the research of the project and the supervision it provides. Dr. D. L. Fisk of Northwestern University visited Kent in October and again in December for research consultation in connection with the project. Fisk will receive partial support from the project during the summer and is expected to join the faculty of the mathematics department at Kent beginning fall quarter of 1967. Professor Byron McCandles, topologist and professor in the department, has expressed an interest in the project work, as has Dr. J. O. Batt, visiting professor from Heidelberg University. Both will possibly receive partial support during part of the summer term. Dr. Chatterji will visit during the summer supported by the department at which time he will teach one or more graduate courses.

At the present time plans are being made for a Symposium on Stationary Stochastic Processes to be held at Kent during the summer of 1967. Several well-known mathematicians and statisticians have expressed plans to attend the symposium, among whom are Professor P. R. Masani, of Indiana University; Professor J. Gani, of the University of Sheffield; Professor V. S. Mandrekar, of the University of Minnesota; and Professor H. Salehi, of Michigan State University.

The principal investigator, jointly with D. L. Fisk, has plans to submit an invited research paper for presentation at the International Statistical Institute meetings in Australia in August-September, 1967.

III. CURRENT RESEARCH ACTIVITIES

Most of the current research activities remain the same as described in the Semiannual Status Report No. 5. At present most of the research work done under the sponsorship of the research grant is being put into a somewhat semifinal form so that within the coming academic year it will be possible to submit a complete report of the research project. Many pieces of work are in the process of being published or are ready to be submitted for publication. All of these will be described in detail in our next two Semiannual reports. Some of the new research which has been undertaken recently is described below with the names of the persons most interested in that work.

<u>1. Covered Spaces</u> [Bhargava, Doyle, Edelman]

By a covered space (X, $\mathcal{H}$) is meant a nonempty set X and a collection $\mathcal{H}$ of subsets of X (called open sets) such that the union of all sets H belonging to $\mathcal{H}$ equals X. The main problem considered in this area is to determine the extent to which covered spaces may be regarded as meaningful generalizations of topological spaces. A point x in X is said to be a limit point of a subset E of X if every member of $\mathcal{H}$ which contains x has a nonempty intersection with E-x. We define the derived set E' to be the set of all the limit points of a set E, and we say a set is closed if it contains all its limit points. We define the closure $\overline{E}$ of a set E to be the union of that set with its derived set. Finally we define the interior of a set E to be the complement of the closure of the complement of the set: $\mathrm{Int}\,(E) = X - \overline{X-E}$. It is then possible to show that these three operations (derived set, closure, interior) satisfy nearly all of the elementary properties which are true of the corresponding operations defined on topological spaces. In particular, each operation satisfies the conditions of a theorem from topology, which states that this operation on subsets of X determines a unique topology on X in which this is the corresponding operation for the topology.

The most fundamental difference so far discovered between covered and topological spaces is the structure of the open sets, or members of the collection of subsets of X. It has been shown that the complement of every union of open sets (and hence, of every single open set) is closed, but the most we can say about the complement of a closed set is that it is the union of open sets, which itself may not belong to H. This distinction is in essence the reason that the interior of a set E is not an open set and hence cannot be characterized as the largest open set contained in E. However, closed sets behave very similarly to the closed sets of a topological space, and it is in fact possible to characterize the closure of a set E as the smallest closed set containing E.

Investigation is continuing along these lines with the intention of introducing such concepts as continuity, separation, and other ideas which may allow signigicant results to be obtained.

2. Extension of Riemannian Geometry [Zaustinsky]

Several approaches to the question of extending the results of Riemannian geometry to more general spaces are being investigated with special interest in problems of a qualitative or global character and which are, therefore, closely related to generalizations of the calculus of variations. Since it is known that a sufficiently regular space must be Riemannian (for instance, F. W. Warner has shown that differentiability of the metric of a Finsler space implies that it is Riemannian), it is natural to try to work in the absence of differentiability hypotheses. We proceed to describe a number of results already obtained and to indicate related problems with which we are now primarily concerned.

A G-space is a complete metric space which satisfies additional hypotheses which guarantee the existence of geodesics with the usual properties. A natural development of the theory, not previously considered, is the treatment of conjugate and cut-points in the absence of differentiability hypotheses. If p is a point of a G-space R, a cut-point of p along a geodesic beginning at p can be defined exactly as in the differentiable case. To define a conjugate-point, let K(p) be a sufficiently small sphere with center p (i.e., a set $\{x \in R \mid d(p,x) = \varepsilon\}$, for some $\varepsilon > 0$). Then $K \times [0,+\infty)$ can be mapped into R by the obvious generalization of the usual exponential map and a conjugate-point can be defined as a singularity of this map (i.e., a point where the map is not locally injective). Among our results obtained so far, are the theorem that a geodesic minimizes arclength locally up to the first conjugate-point, as well as many of the qualitative results of the classical theory, as developed by Myers, Whitehead, Klingenberg, and others, describing the cut and conjugate loci. The question whether a geodesic ceases to minimize acrlength after the first conjugate-point appears to be much more difficult and remains open.

Another line of investigation concerns the implications of restrictions on the curvature for the topological character of the space. Kann has introduced the notion of curvature greater than a constant K for a two-dimensional G-space and proved the Theorem of Bonnet that a two-dimensional space with curvature greater than $K > 0$ is compact. We have obtained most of the results complementary to those of Kann (for instance, the Theorem of Pogorelov that the shortest closed geodesic in a simply connected space with curvature less than or equal to L, has length at least equal to $2\pi/\sqrt{L}$) but, unfortunately, most of our results of this type are restricted to dimension 2. An important open problem is therefore to find a better definition of curvature which will permit carrying over this type of result to higher dimensions.

In a quite different direction, we have attempted to treat Finsler spaces with the usual differentiability hypotheses by means of approximation by means of Riemannian spaces. In this way, we have been able to extend the following very deep Theorem of Löwner to two-dimensional Finsler spaces: Löwner's Theorem states that if ℓ is the length of the shortest closed geodesic on a two-dimensional Riemannian manifold R of the topological type of the torus, then $\ell^2 \leq 2/\sqrt{3} \cdot A$, where A is the area of R. Furthermore, the equality can only hold if R is a certain identification space of the plane. Several interesting questions related to the Löwner Theorem remain open. The first has to do with the approximation by means of Riemannian spaces. We use a quadratic form F on the space, defined by means of the Finsler metric, which we are only able to show is continuous. Our proof then depends on a differentiable approximation to the quadratic form F. Question: Is F actually differentiable and, therefore, already Riemannian, in fact? But much more interesting is the question of finding any generalization of Löwner's Theorem to higher dimensions.

We have been working on the following Conjecture: Consider a Riemannian metric on the 4-torus T^4. Let S be the smallest 2-dimensional homology class and V the volume of the torus. Then, $S^2 \leqq kV$, where k is a constant independent of the Riemannian metric. Furthermore, equality can hold only for a certain indentification space of the euclidean space E^4 with the usual flat metric.

Our most immediate research interests are, of course, concerned with putting our results obtained so far into publishable form. At the moment, we are thinking in terms of organizing them into three papers: 1) The treatment of conjugate and cut-points in the absence of differentiability hypotheses, 2) Spaces with positive curvature, and 3) The approximation of Finsler metrics by means of Riemannian metrics.

3. Maximal Functions over Partitions [Bhargava, Chatterji, Smucker]

An ordered partition of length k of the positive integer n is defined to be an ordered k-tuple of positive integers whose sum is n, $n \geqq k \geqq 2$; and the set of all ordered partitions of length k of the positive integer n will be denoted by π_n^k. If an element π in π_n^k is given by $\pi = (x_1, x_2, \ldots, x_k)$, the number $\Sigma(\pi,j)$ is defined by $\Sigma(\pi,j) = \sum_{i=1}^{k-j} x_i \, x_{i+1} \cdots x_{i+j}$, where j is an integer such that $1 \leqq j \leqq k-1$. The number σ_j is defined to be $\sigma_j = \max \Sigma(\pi,j)$. It is proposed to find an explicit formula for $\pi \varepsilon \pi_n^k$, σ_j in terms of k, n and j.

For the special case $j = 1$, which essentially arose from certain work of S. D. Chatterji, σ_1 has been found by A. Meir (University of Alberta) to be

$$\sigma_1 = \begin{cases} \left[\frac{n}{2}\right] \cdot \left[\frac{n+1}{2}\right], & k = 2, 3 \\ 2n-k-1 + \left[\frac{n-k}{2}\right] \cdot \left[\frac{n-k+1}{2}\right], & k \geqq 4. \end{cases}$$

Brackets denote the greatest integer function. The case $j \geqq 2$ is under investigation.

Furthermore, if the variables $x_1, x_2, \cdot \cdot \cdot, x_k$ are permitted to be non-negative real numbers instead of positive integers, the above definitions all still have meaning. It is proposed to find σ_j as a function of k, n and j in this case also.

Resumé : Eugene M. Zaustinsky

(Consultant-visitor)

Date and Place of Birth: October 19, 1926; Battle Creek, Michigan

Education Experience:

Degrees:	Institution:
B.A.	University of California (Los Angeles)
M.A.	University of Southern California
Ph.D.	University of Southern California

Teaching and Training:

Assistant Professor (1957-1958)	San Jose State College, California
Assistant Professor (1958-1961)	University of California (Santa Barbara)
Assistant Research Mathematician (1961-1963)	University of California (Berkeley)
Associate Professor (1963-)	State University of New York (Stony Brook)

Field of Interest: Differential geometry

Papers:

"Spaces with Non-symmetric Distance", Memoirs American Mathematical Society, 1959.

"Extremals on Compact E-Surfaces", Transactions of American Mathematical Society, (1962).

"G-Surfaces with Positive Curvature", to appear.